

Sample Problem Sheet

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1. Differentiate from first principles $f(x) = c$ where c is a constant.

Solution:

$$\begin{aligned}\frac{df}{dx} &= \lim_{\Delta x \rightarrow 0} \frac{c - c}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} 0 \\ &= 0\end{aligned}$$

2. Differentiate from first principles $f(x) = \sqrt{x}$

Solution:

$$\begin{aligned}\frac{df}{dx} &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{(\sqrt{x + \Delta x} - \sqrt{x})(\sqrt{x + \Delta x} + \sqrt{x})}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})} \\ &= \lim_{\Delta x \rightarrow 0} \frac{x + \Delta x - x}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})} \\ &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} \\ &= \frac{1}{2\sqrt{x}}\end{aligned}$$

3. Differentiate the following functions:

(a) $f(x) = 2x^2 + 3x - 1$

Solution: $f'(x) = 4x + 3$

(b) $f'(x) = -3 \cos(-3x)$