

Testing amsrefs with the hyperref package

MJD

June 24, 2011

The following examples are derived from *Homology manifold bordism* by Heather Johnston and Andrew Ranicki (Trans. Amer. Math. Soc. **352** no 11 (2000), PII: S 0002-9947(00)02630-1).

4 Homology manifold bordism

The results of Johnston [5] on homology manifolds are extended here. It is not possible to investigate transversality by geometric methods—as in [5] we employ bordism and surgery instead.

The proof of transversality is indirect, relying heavily on surgery theory—see Kirby and Siebenmann [7, III, §1], Marin [8] and Quinn [11]. We shall use the formulation in terms of topological block bundles of Rourke and Sanderson [12].

Q is a codimension q subspace by Theorem 4.9 of Rourke and Sanderson [12]. (Hughes, Taylor and Williams [4] obtained a topological regular neighborhood theorem for arbitrary submanifolds ...)

Wall [13, Chapter 11] obtained a codimension q splitting obstruction ...

... following the work of Cohen [2] on PL manifold transversality.

In this case each inverse image is automatically a PL submanifold of codimension σ (Cohen [2]), so there is no need to use s -cobordisms.

Quinn [10, 1.1] proved that ...

Theorem 4.1 (The additive structure of homology manifold bordism, Johnston [5])

...

For $m \geq 5$ the Novikov-Wall surgery theory for topological manifolds gives an exact sequence (Wall [13, Chapter 10]).

The surgery theory of topological manifolds was extended to homology manifolds in Quinn [9, 10] and Bryant, Ferry, Mio and Weinberger [1].

The 4-periodic obstruction is equivalent to an m -dimensional homology manifold, by [1].

Thus, the surgery exact sequence of [1] does not follow Wall [13] in relating homology manifold structures and normal invariants.

... the canonical TOP reduction ([3]) of the Spivak normal fibration of M ...

Theorem 4.2 (Johnston [5]) ...

Actually [5, (5.2)] is for $m \geq 7$, but we can improve to $m \geq 6$ by a slight variation of the proof as described below.

(This type of surgery on a Poincaré space is in the tradition of Lowell Jones [6].)

References

- [1] J. Bryant, S. Ferry, W. Mio, and S. Weinberger, *Topology of homology manifolds*, Ann. of Math. **143** (1996), 435–467. MR97b:57017 ↑1
- [2] M. Cohen, *Simplicial structures and transverse cellularity*, Ann. of Math. **85** (1967), 218–245. MR35:1037 ↑1
- [3] S. Ferry and E. K. Pedersen, *Epsilon surgery theory I*, Novikov conjectures, index theorems and rigidity, vol. 2 (Oberwolfach, 1993), 1995, pp. 167–226. MR97g:57044 ↑1
- [4] B. Hughes, L. Taylor, and B. Williams, *Manifold approximate fibrations are approximately bundles*, Forum Math. **3** (1991), 309–325. MR92k:57040 ↑1
- [5] H. Johnston, *Transversality for homology manifolds*, Topology **38** (1999), 673–697. MR99k:57048 ↑1
- [6] L. Jones, *Patch spaces: a geometric representation for Poincaré spaces*, Ann. of Math. **97** (1973), 306–343. 102, 183–185 (1975) MR47:4269; MR52:11930. ↑1
- [7] R. Kirby and L. Siebenmann, *Foundational essays on topological manifolds, smoothings, and triangulations*, Ann. of Math. Study, vol. 88, Princeton University Press, 1977. MR58:31082 ↑1
- [8] A. Marin, *La transversalité topologique*, Ann. of Math. **106** (1977), 269–293 (French). MR57:10707 ↑1
- [9] F. Quinn, *Resolutions of homology manifolds, and the topological characterization of manifolds*, Invent. Math. **72** (1983), 264–284. Corrigendum **85** (1986) 653. MR85b:57023, MR87g:57031 ↑1
- [10] ———, *An obstruction to the resolution of homology manifolds*, Michigan Math. J. **34** (1987), 284–291. MR88j:57016 ↑1
- [11] ———, *Topological transversality holds in all dimensions*, Bull. Amer. Math. Soc. **18** (1988), 145–148. MR89c:57016 ↑1
- [12] C. P. Rourke and B. J. Sanderson, *On topological neighbourhoods*, Compositio Math. **22** (1970), 387–425. MR45:7720 ↑1
- [13] C. T. C. Wall, *Surgery on compact manifolds*, 2nd ed., Academic Press, 1970. ↑1